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Approach to Discrete Optimization Under Uncertainty: The Population-Based Sampling Genetic Algorithm

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This paper presents the population-based sampling genetic algorithm, which allows for discrete design optimization under uncertainty. The population-based sampling approach uses the genetic algorithm population to provide samples for the probabilistic evaluation of aggregate uncertain constraint or objective functions. In population-based sampling, large numbers of samples are accumulated to evaluate the fitness values of "good" designs during the genetic algorithm run, whereas the computational cost spent on designs with "poor" fitness is minimal. Using Monte Carlo sampling with a genetic algorithm for optimization under uncertainty is a currently accepted approach; however, this approach incurs a large computational cost. In this paper, the genetic algorithm with population-based sampling generates solutions to a discrete optimization problem under uncertainty associated with a commercial satellite design that was solved in previous work via a genetic algorithm with Monte Carlo sampling. The genetic algorithm with population-based sampling and genetic algorithm with Monte Carlo sampling approaches are compared in terms of efficiency (computational cost) and effectiveness (solution quality). The comparison also examines the scalability of the algorithms' performance when solving three additional problem sizes. Furthermore, two population-based sampling variants are introduced, namely, the variable population-based sampling approach, which combines the concepts of population-based sampling and Monte Carlo sampling, and the generalized population-based sampling approach, which removes the restriction in population-based sampling that the uncertain parameters associated with the design variables all have Gaussian probability distributions.

Nomenclature

c	=	penalty multiplier
$E()$	=	expected value of
f	=	fitness function
G	=	uncertain inequality constraint
g	=	deterministic inequality constraint
M	=	mass
N_{samples}	=	number of samples
$P()$	=	probability of
R	=	reliability
$\boldsymbol{x}$	=	design vector
λ	=	failure rate
μ	=	mean
$\boldsymbol{\xi}$	=	uncertain parameters vector
σ	=	standard deviation
Φ	=	Gaussian probability density function
ϕ	=	objective function

I. Introduction

PROBABILISTIC decision making is a branch of statistics known as statistical inference that is concerned with the formulation of generalizations about and the prediction of uncertain characteristics (usually performance metrics) of systems on the basis of limited information. This involves drawing conclusions about a set of finite or infinite data (called a population) from a small subset of observed values of the population. This small subset of data is a "sample," and drawing this subset from the population is "sampling." The sample should be a good representation of the entire population. To avoid bias in conclusions drawn about a system's uncertain characteristics, samples should be randomly drawn from the population representing those characteristics. The estimates of the uncertain system performance metrics will increase in accuracy with increasing sample size.

The most common way to express uncertainty about an estimate of a system performance metric is to define an interval or range of values, based on the observed values in a sample, in which the performance metric is likely to be. The range of values representing the estimate of uncertain characteristics is the "confidence interval." Given a fixed sample size, a wider range indicates a larger probability that the uncertain metric belongs to the confidence interval. This probability is the "confidence level." For a predetermined confidence interval, confidence levels increase with increasing sample size, because more gathered information about an uncertain characteristic allows statistical inferences with higher confidence.

For system design under uncertainty, the probabilistic evaluation of uncertain aggregate performance metrics typically involves several uncertain parameters. A large sample size provides better accuracy in predicting uncertain metrics, although a large sample size requires a large number of system performance analyses, which can be computationally expensive.

Complex systems have many design variables that can be combined into numerous possible concepts. Optimization approaches can assist the search for the best design concept(s); optimization involves performance evaluation of many possible solutions. When using probabilistic analysis with an optimization approach, the computational cost could become prohibitive, because the probabilistic evaluation of each design solution encountered in the optimization process must use a large sample size to estimate performance with high confidence levels. The challenge is that a large number of samples are needed to have reasonable estimates, but the number of samples must be kept small to enable an optimal search.

Previous work by the authors [1] described a genetic algorithm (GA) with Monte Carlo sampling (MCS), a traditional probabilistic sampling approach, and applied this GA-MCS approach to solve a

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commercial communication satellite design problem. This example is a discrete optimization problem involving uncertain reliability metrics. Also, this previous research compares the GA-MCS approach to a deterministic margin-based (DMB) approach, which is an analog to a safety-factor approach to address uncertainty. This comparison demonstrated the advantage of probabilistic approaches in spacecraft design. The GA-MCS approach found low mass spacecraft design solutions that satisfy reliability constraints at high confidence levels. However, the GA-MCS approach incurred a very large computational cost, and the implementation limited each function evaluation to only 500 samples for probabilistic evaluation. In contrast, the GA-DMB approach provided satellite designs that are more than 1000 kg heavier than those obtained by the GA-MCS. The GA-DMB approach, however, required 3 orders of magnitude less computational effort than the GA-MCS approach [1].

The contribution of this paper is the presentation of the population-based sampling (PBS) approach, which combines the advantages of both the MCS and the DMB approaches by exploiting the populations of solutions generated in a GA run. This paper also presents variations of the PBS approach and discusses these as ways to improve prediction accuracy of uncertain parameters and to reduce total computational effort. The GA-PBS approach and its variants provide means for discrete optimization under uncertainty that provide similar solutions at a greatly reduced computational cost than current methods that rely upon MCS to address uncertainty. This paper uses a satellite design problem to illustrate the GA-PBS approach, but the approach is general and applicable to discrete optimization problems that include uncertain objective and/or constraint functions.

II. Commercial Satellite Design Example

Throughout this paper, the communication satellite reliability-based design optimization problem of [1] is used in the discussion. The problem has 27 discrete variables, which appear in Table 1.

Fourteen categorical variables represent the technology choice of various components and subsystems (e.g., the use of chemical or electric propulsion for stationkeeping) and the choice of the launch vehicle. The remaining ordinal variables describe redundancy levels (e.g., zero, one, two, or four additional high-power amplifiers per set of 12). Because this is a discrete problem, the GA is a well-suited search method.

This problem seeks to minimize the satellite's launch mass as a surrogate for cost. Three deterministic constraints, $g_1 - g_3$, limit the mass and dimensions of the spacecraft based upon the chosen launch vehicle's payload fairing and load to orbit capacity. Three additional constraints, $G_4 - G_6$, enforce limits on the launch vehicle reliability, the satellite's payload reliability, and the total spacecraft reliability. These reliability constraints are probabilistic, because the published failure rates and reliability values of the various spacecraft components are treated as uncertain properties. As a result, the reliability constraints enforce the fact that the computed reliability meets or exceeds a defined reliability measure with a high probability or high confidence level. The use of capital letters for these functions indicates their uncertain nature. A combined satellite sizing and reliability tool, which is more fully described in [1,2], computes values for the dimensions, masses, and reliabilities. Equations (1–7) summarize the problem statement.

$$\text{Minimize} \quad \phi(\boldsymbol{x}) = M_{\text{spacecraft}} \tag{1}$$

$$\text{Subject to} \quad g_1(\boldsymbol{x}) = \frac{\text{solar array panel height}}{\text{launch vehicle fairing height}} - 1 \leq 0 \tag{2}$$

$$g_2(\boldsymbol{x}) = \frac{\text{radiator height}}{\text{launch vehicle fairing height}} - 1 \leq 0 \tag{3}$$

$$g_3(\boldsymbol{x}) = \frac{M_{\text{spacecraft}}}{\text{allowable launch vehicle lift mass}} - 1 \leq 0 \tag{4}$$

$$G_4(\boldsymbol{x}, \boldsymbol{\xi}) = 1 - \frac{P(R_{\text{launcher}}(\boldsymbol{x}, \boldsymbol{\xi}) \geq 0.90)}{\text{confidence level}} \leq 0 \tag{5}$$

$$G_5(\boldsymbol{x}, \boldsymbol{\xi}) = 1 - \frac{P(R_{\text{payload}}(\boldsymbol{x}, \boldsymbol{\xi}) \geq 0.99)}{\text{confidence level}} \leq 0 \tag{6}$$

$$G_6(\boldsymbol{x}, \boldsymbol{\xi}) = 1 - \frac{P(R_{\text{spacecraft}}(\mathbf{x}, \boldsymbol{\xi}) \geq 0.95)}{\text{confidence level}} \leq 0 \tag{7}$$

Using the GA as the optimization method allows for discrete design variables to be encoded in the binary chromosomes representing individual satellite designs. To perform the selection operator that permits designs with good design characteristics to

Table 1 Satellite problem design variables

	Design variable	Discrete values
1	Launch vehicle choice from 8 options	Ariane 4, Ariane 5, Proton, Delta, Atlas, Long March, Sea Launch, or H2A
2–9	HPA[a] type for the 8 C-band Txs[b]	TWTA[c] or SSPA[d]
10	Solar array cell type	GaAs single junction, GaAs multijunction, Si thin, Si normal, or hybrids of Si and GaAs
11	Battery cell type	NiCd or NiH_2
12	N/S[e] thermal radiator panels coupling	Yes or no
13	N/S STK[f] thruster technology	Xenon plasma, arcjets, bipropellant, or hydrazine
14	E/W[g] STK thruster technology	Bipropellant or hydrazine
15	Redundancy level for the Ku-band Tx	0, 1, 2, or 4 redundant HPAs for 12 operating HPAs
16–22	Redundancy level for 7 C-band Txs	0, 1, 2, or 4 redundant HPAs for 12 operating HPAs
23	Redundancy level for last C-band Tx	0 or 1 redundant HPAs for 2 operating HPAs
24	Propulsion subsystem redundancy level	No redundancy or duplicate redundancy
25	ADCS[h] redundancy level	No redundancy or duplicate redundancy
26	TCR[i] subsystem redundancy level	No redundancy or duplicate redundancy
27	Solar array redundancy level	0, 2, 4, or 6% of solar array area

[a]High-power amplifier.
[b]Transponders.
[c]Traveling wave tube amplifier.
[d]Solid state power amplifier.
[e]North/South.
[f]Stationkeeping.
[g]East/West.
[h]Attitude determination and control subsystem.
[i]Telemetry, command, and ranging.

survive and serve as "parents" for a subsequent generation of designs, the GA requires a single fitness function. This fitness function uses a penalty approach to combine the objective and constraint functions. With three uncertain constraint functions, the fitness function becomes uncertain as shown in Eq. (8):

$$f(\boldsymbol{x}, \boldsymbol{\xi}) = \phi(\boldsymbol{x}) + \sum_{i=1}^{3} c_i \max[0, g_i(\boldsymbol{x})] + \sum_{j=4}^{6} c_j \max[0, G_j(\boldsymbol{x}, \boldsymbol{\xi})] \tag{8}$$

III. Population-Based Sampling (PBS)

PBS was first suggested by Crossley in 1999 [3], and it was subsequently mathematically formulated and applied to the spacecraft deign problem in this research [2]. PBS makes use of the large number of designs evaluated by the GA to provide samples for probabilistic evaluation of designs that repeatedly appear in the GA population. As the GA's search progresses, the population congregates in areas of the design space associated with good fitness values. Frequently, the same design description appears in several consecutive generations and usually appears multiple times in the same generation as the algorithm reaches the end of its search. To provide samples of the uncertain parameters in this approach, each design chromosome in the population is decoded into its corresponding deterministic design variables. Random values are sampled and assigned to each uncertain parameter from its associated probability density function (PDF). The calculation of the objective and constraint functions of each individual design uses the combination of deterministic design variables and the sampled uncertain parameters.

The values of the fitness function are uncertain; that is, two individual designs sharing the same deterministic design variables will likely have different fitness values because each design has a different sample set assigned to its uncertain parameters. In PBS, every individual evaluated by the GA is stored and each of these individuals has a different set of samples for its uncertain parameters. In each generation, the expected fitness value of a given design is calculated as the mean of all the fitness values of all individuals sharing the same deterministic design variables. The individuals used in this calculation are taken from all previous generations, not just the current generation. Calculating statistical performance measures other than the expected value, such as standard deviation, is possible.

Initially, the GA generates a random population; therefore, in early generations, there are only few, if any, designs that share the same deterministic design variables. In this case, the expected fitness value will be calculated based on one or very few sample sets for the uncertain parameters. Later, as the GA population congregates around better designs, many individuals will share the same design variables; therefore, each design will have many samples for its uncertain parameters, which better represents the uncertainty in the expected fitness values.

A design that appears 500 times in a GA-PBS run will have the same accuracy of fitness prediction as if this same design was evaluated using 500 samples in an MCS approach. The difference is that in using MCS with the GA, every single design that appears during the GA run will have 500 (or any other preselected number) samples used for its fitness evaluation. In PBS, good designs will keep reappearing in the genome pool, and only these will accumulate the required number of samples for their fitness evaluation. Poor designs will accumulate few, if any, samples for the evaluation of their fitness values. This is much like conducting a deterministic search during the early generations of the GA, then providing a probabilistic search near the end of the GA run. This will greatly reduce the total computational effort required to complete a GA search for design under uncertainty.

A. Confidence Level Constraint Handling in PBS

A confidence level constraint provides a lower limit for the probability that an aggregate uncertain performance value satisfies a specified performance constraint. In the satellite design discrete optimization problem example, one of the aggregate uncertain system performance metrics is the total spacecraft reliability. Spacecraft reliability is assembled from the reliability values of the various components and subsystems. These reliability values are uncertain, because components and subsystems failure rates are uncertain. The confidence level constraint imposes a limit on the probability of meeting the reliability requirement instead of imposing a constraint on the expected value of spacecraft reliability. This reflects that it is not sufficient to obtain a design solution with a high expected reliability value; rather, it is crucial that the reliability requirement is met with a high probability, that is, a high confidence level is met. Equation (7) demonstrates the mathematical expression of a confidence level constraint. In Eq. (7), the probability that the spacecraft reliability exceeds 0.95 must be greater than or equal to the confidence level value. If the confidence level value is chosen to be 99%, then this constraint would be satisfied if the probability that the spacecraft reliability meets or exceeds 0.95 is 99% or higher.

In the traditional GA-MCS approach, many samples are used to evaluate each single design, and so probabilities can be calculated using counting; therefore, confidence level constraints can be imposed directly as shown in Eq. (7). However, in the PBS approach, designs evaluated at the beginning of the GA run will have very few copies in either the current or previous generations. A design appearing for the first time will have one copy and only one sample set of the uncertain parameters and, as a result, the calculated probability value is either 0 or 1. Constraints posed like Eq. (7) will be meaningless for designs with very few samples.

Instead, a "surrogate confidence level constraint" formulation can calculate probabilities of success, that is, feasibility with respect to reliability limits, using approximate distributions for the uncertain aggregate performance metric. The sample sets that a design accumulates during the GA run provide a measured expected value (or mean) μ_m and a measured standard deviation σ_m for the uncertain value. This uncertain performance metric is assumed to follow a Gaussian distribution with a mean and standard deviation equal to μ_m and σ_m, respectively. Because the properties of a Gaussian distribution are well known, a margin-like constraint can limit the expected value as a surrogate for enforcing the confidence level constraints.

In the spacecraft design problem, the reliabilities of the various components and subsystems were assumed to follow pseudo-Gaussian distributions. For a variety of reasons, data describing the actual probability distribution of component and subsystem reliabilities were not available; in most cases, only a single published reliability value is provided. The published component reliability value served as the mean value of a Gaussian distribution, and an assumed percentage of this published value served as the standard deviation of the distribution. It is possible to sample a reliability value greater than one from the resulting Gaussian distribution. Because components cannot have reliabilities that exceed 100%, if a reliability value exceeding one was sampled from the distribution, this value was ignored and another sample was taken. This truncates the infinite right-hand tail of the Gaussian distribution to end at 100%, and it ensures that aggregate reliabilities computed for the spacecraft or payload cannot exceed 100%. The resulting pseudo-Gaussian distributions have a form similar to the example in Fig. 1. Details on the values used for component and subsystem reliabilities are available in [1,2].

Because the component and subsystem reliability distributions are nearly Gaussian, it can be assumed—but should be confirmed in a postprocessing procedure—that the aggregate system level reliability follows a Gaussian distribution. During a GA run, a design solution accumulates a number of samples N_{samples}, from which Eqs. (9) and (10) can calculate a measured mean value μ_m and a measured standard deviation σ_m of reliability that describe the assumed Gaussian distribution.

$$\mu_m = E(R_{\text{spacecraft}}) = \frac{\sum_{j=1}^{N_{\text{samples}}} R_{\text{spacecraft}_j}}{N_{\text{samples}}} \tag{9}$$

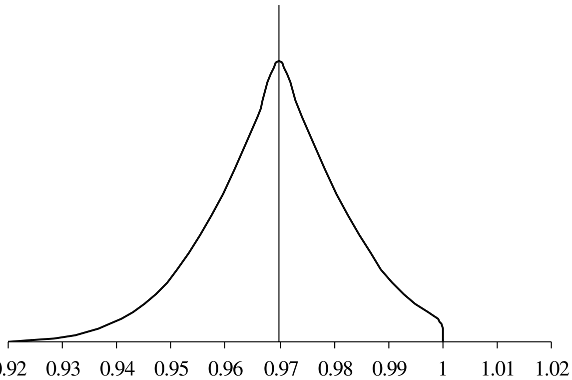

Fig. 1 Example pseudo-Gaussian distribution of component reliability (here $\mu_{\text{component}} = 0.97$ and $\sigma_{\text{component}} = 0.0167$).

$$\sigma_m = \sqrt{E\left(R^2_{\text{spacecraft}}\right) - E^2\left(R_{\text{spacecraft}}\right)} \tag{10}$$

With the Gaussian distribution assumption, the surrogate confidence level constraint uses the inverse normal density function Φ^{-1} as in Eq. (11).

$$\frac{\mu_m - 0.95}{\sigma_m} \geq \Phi^{-1}(\text{confidence level}) \tag{11}$$

If the values of μ_m and σ_m satisfy the inequality, the percentage of a Gaussian distribution that exceeds the reliability value of 0.95 is greater than or equal to the confidence level limit. The advantages of the surrogate constraints are that values of Φ^{-1} are easily obtained (see, for example, [4]) and that μ_m and σ_m have continuous values with a small number of samples.

Figure 2 illustrates this concept. If the probability of satisfying the reliability constraint is 99%, then 99% of the reliability values in the assumed Gaussian distribution should be greater than or equal to a value of 0.95. The area under the Gaussian distribution to the right of the hashed line corresponds to 99% of the total area under the curve.

For a numerical illustration, assume that the measured mean and standard deviation of spacecraft reliability for a given design are 0.97 and 0.005, respectively. These values result in $(\mu_m - 0.95)/\sigma_m = 4$. For a confidence level of 99%, the inverse normal distribution value $\Phi^{-1}(0.99) = 2.326$. In this case, Eq. (11) is satisfied ($4 \geq 2.326$), so the fitness function for this design includes no penalty for this constraint. Consider a different design with $\mu_m = 0.96$ and $\sigma_m = 0.007$. This design's expected reliability value exceeds the limit ($0.96 \geq 0.95$), but the assumed Gaussian distribution for this design has only 98.9% of the area to the right of the limiting reliability value of 0.95. As a result, Eq. (11) is not satisfied, $(\mu_m - 0.95)/\sigma_m = 2.286$. The violation is small and the fitness function for this design would include a small penalty.

Equations (12–14) display the surrogate confidence level constraint formulations for the spacecraft problem. The payload and launch vehicle reliability constraints also use the surrogate confidence level approach. The surrogate constraint functions are negative valued when satisfied.

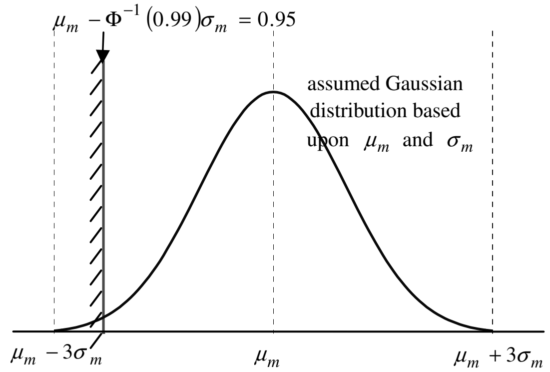

Fig. 2 Assumed Gaussian distribution with 0.95 reliability limit at a 99% confidence level.

This approach provides useful constraint function values with few samples. A design with no other copies in the population has only one sample set from the probability distributions used in Eqs. (12–14). With one sample, the surrogate constraint is treated like a deterministic constraint, because the mean μ_m is computed using only one sample and the standard deviation σ_m is zero. For example, the surrogate constraint G_6 would be treated as $g_6 = 1 - (\mu_m/0.95)$ with only one sample.

B. Fitness Evaluation Procedure in PBS

The following procedure computes the fitness value for each individual. For a given population, each individual (chromosome string) is decoded into the corresponding design vector $\boldsymbol{x}$ and is assigned a vector of uncertain parameters $\boldsymbol{\xi}$, based upon one set of samples from the probability distributions of component and subsystem reliabilities. The objective $\phi(\boldsymbol{x})$, deterministic constraints $g_1(\boldsymbol{x})$, $g_2(\boldsymbol{x})$, and $g_3(\boldsymbol{x})$, and the uncertain reliability values $R_{\text{launcher}}(\boldsymbol{x},\boldsymbol{\xi})$, $R_{\text{payload}}(\boldsymbol{x},\boldsymbol{\xi})$, and $R_{\text{spacecraft}}(\boldsymbol{x},\boldsymbol{\xi})$ are calculated. Every previous individual evaluated by the GA has been stored (for the initial generation, there are no stored individuals), and each of these individuals also has one sample of the uncertain parameter set $\boldsymbol{\xi}$. The current population is added to the stored individuals and this group is sorted into equivalence classes using the deterministic design variable vector $\boldsymbol{x}$. Two designs are equivalent if they have the same $\boldsymbol{x}$. For all N_{samples} designs sharing the same design vector $\boldsymbol{x}$, Eqs. (9) and (10) determine the values of the measured means μ_m and measured standard deviations σ_m of the assumed aggregate reliability distributions; then Eqs. (12–14) provide values of the surrogate constraints G_j.

The fitness function incorporates values for the deterministic constraints g_i and the surrogate confidence level constraints G_j. In this work, a linear exterior penalty function formulation, shown in Eq. (8), accounts for violated constraints.

Once Eq. (8) computes a fitness value for each individual, the GA operators of selection, crossover, and mutation form the next generation. This process then repeats each generation, and each new individual chromosome is decoded to the corresponding $\boldsymbol{x}$ and is assigned a $\boldsymbol{\xi}$ based upon one sample from the probability distributions. Values for ϕ, g_i, and for R_{launcher}, R_{payload}, and $R_{\text{spacecraft}}$ are determined for each new individual. These individuals are added to the stored list of individuals, the stored list is grouped into

$$G_4(\boldsymbol{x},\boldsymbol{\xi}) = 1 - \frac{\mu_m(R_{\text{launcher}}(\boldsymbol{x},\boldsymbol{\xi})) - \Phi^{-1}(\text{confidence level}) \times \sigma_m(R_{\text{launcher}}(\boldsymbol{x},\boldsymbol{\xi}))}{90\%} \leq 0 \tag{12}$$

$$G_5(\boldsymbol{x},\boldsymbol{\xi}) = 1 - \frac{\mu_m(R_{\text{payload}}(\boldsymbol{x},\boldsymbol{\xi})) - \Phi^{-1}(\text{confidence level}) \times \sigma_m(R_{\text{payload}}(\boldsymbol{x},\boldsymbol{\xi}))}{99\%} \leq 0 \tag{13}$$

$$G_6(\boldsymbol{x},\boldsymbol{\xi}) = 1 - \frac{\mu_m(R_{\text{spacecraft}}(\boldsymbol{x},\boldsymbol{\xi})) - \Phi^{-1}(\text{confidence level}) \times \sigma_m(R_{\text{spacecraft}}(\boldsymbol{x},\boldsymbol{\xi}))}{95\%} \leq 0 \tag{14}$$

equivalence classes, and then the current population's individuals are assigned fitness values.

In early generations, there is little chance that two or more individuals will have the same design variable vector $\boldsymbol{x}$. In this case, $N_{\text{samples}} = 1$ in Eq. (9) and the surrogate constraints act like deterministic constraints as discussed previously. As the GA search continues and the population congregates in areas of good fitness, it becomes increasingly likely that two or more individuals will share the same $\boldsymbol{x}$. In generations at the end of the GA run, a large number of designs will share the same $\boldsymbol{x}$, but each design will have different samples of $\boldsymbol{\xi}$. From these points, the measured means μ_m and standard deviations σ_m of the reliability distributions improve in accuracy, and the constraints to address uncertainty are incorporated with no additional function evaluations. The designs evaluated during the GA run can be "postprocessed" after the run is complete to obtain a probability measure to validate the assumption that the aggregate reliability distributions in the surrogate constraints are Gaussian.

IV. Comparison of PBS and MCS

A quick literature survey showed that both engineering and nonengineering applications of the GA for optimization under uncertainty have generally implemented the expensive MCS approach or other stratified sampling methods. Table 2 summarizes a selected set of these research efforts in chronological order. This list is not exhaustive but is meant to present applications from several disciplines that combine MCS with GA. Based on this literature summary, using MCS for each function evaluation appears to be the state of the practice, if not the state of the art, for optimization under uncertainty using GAs.

In [1], a comparison between the GA-MCS approach and a GA-DMB approach using the spacecraft design problem concluded that although the GA-MCS approach generates considerably lower mass spacecraft designs that satisfy the reliability constraints at high confidence levels, the associated computational cost is prohibitive. This computational cost could discourage practitioners from implementing probabilistic approaches in system design optimization. A few research efforts (see, for example, [5,6]) have been proposed to decrease the computational cost of GA-MCS and robust optimal design approaches in general, but the focus has been on small problems with continuous design variables only.

The major objective of the work in this paper is to formulate and then apply the GA-PBS approach as a tool for optimization of discrete problems under uncertainty with significantly lower computational cost than the GA-MCS approach. This section is organized in two parts. The first part compares the results of the GA-PBS approach to that of the GA-MCS approach in terms of effectiveness (solution quality) and efficiency (computational cost). The second part discusses the results generated by the PBS approach in detail.

Table 2 Summary of representative research efforts implementing traditional sampling approaches with heuristic search methods for design optimization under uncertainty

Publication year (Ref. [no.])	Application	Optimization method	Sampling method
1995 [7]	Personal computer reliability	GA	LHS[a]
1996 [8]	Spacecraft bus reliability	SA[b]	MCS
1998 [9]	Communication network reliability	GA	MCS
2000 [10]	Plant layout with uncertain economic and safety metrics	GA	MCS
2000 [11]	Water quality reliability	GA	MCS
2001 [12]	Supply chain with stochastic demand	GA	MCS
2001 [13]	Energy market stochastic bidding	GA	MCS
2002 [14]	Aircraft engine technology risk	GA	MCS
2002 [15]	Workspace design with uncertain robot configurations	GA	MCS

[a]Latin hypercube sampling.
[b]Simulated annealing.

A. Comparing PBS with MCS

To compare PBS with MCS, the GA was run using the two approaches until meeting comparable stopping criteria. In the MCS approach, a stopping criterion halts the GA if the best fitness value does not change more than ± 0.01 kg for 10 consecutive generations. The fitness evaluation of each design generated in the GA run uses 500 samples in the MCS approach. In the PBS approach, a two-part stopping criterion halts the GA if the best fitness value does not change more than ± 0.01 kg for 10 consecutive generations and if the design associated with this fitness accumulates at least a specified number of samples. For the comparison with the MCS runs, this number is 500 so that the prediction accuracy of the uncertain metrics for the final design found by the PBS approach is the same as for any design evaluated using the MCS approach. For both the MCS and PBS runs, the GA used five different confidence levels, 70, 80, 90, 92.5, and 99%, in the probabilistic constraints. The 92.5% confidence level is the highest confidence level for which the DMB approach could produce a feasible design in a previous study for the spacecraft design problem [1]. At each confidence level, the GA-MCS and GA-PBS were run 10 times with different initial seeds to assess repeatability of the results.

A summary of results from coupling the GA with MCS and PBS for the spacecraft design problem appear in Figs. 3 and 4 and in Tables 3 and 4. Figure 3 displays a bar chart of the mean values of the lowest spacecraft launch mass obtained from 10 runs at each of the five confidence levels; the minimum and maximum values of the best mass value from the 10 runs at each confidence level appear as error bars atop of the columns. Given the scale of the plot, the variations in the resulting best spacecraft mass over 10 runs at each confidence level are small, and, in some cases, all 10 runs provided the same solution. This consistency provides assurance that the GA may have arrived at or near the global optimal solutions in all those runs; however, global optimality cannot be proved for this problem without enumeration. The mean results from both the GA-MCS and GA-PBS runs are very similar.

Figure 4 uses a logarithmic scale to show the mean computational cost of 10 runs at each of the five confidence levels for both approaches; each function evaluation here is one run of the spacecraft sizing code. Again, the minimum and maximum computational costs are shown using error bars; this variation in cost between individual runs is not very significant. Figure 4 shows that GA-PBS uses 2 to 3 orders of magnitude fewer function evaluations to obtain solutions for this problem.

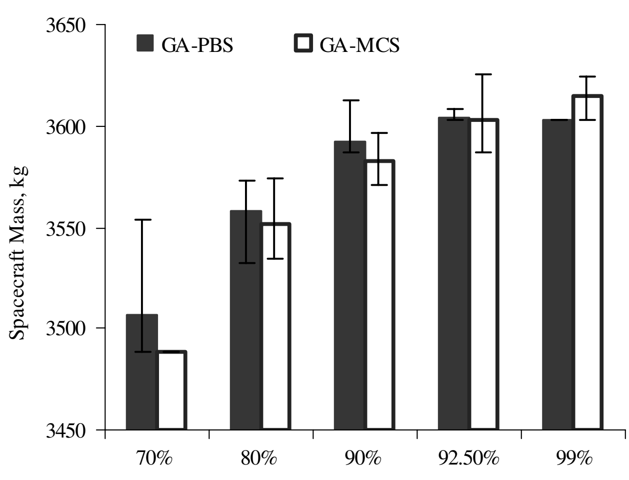

Fig. 3 Lowest spacecraft mass found using MCS and PBS at various confidence levels.

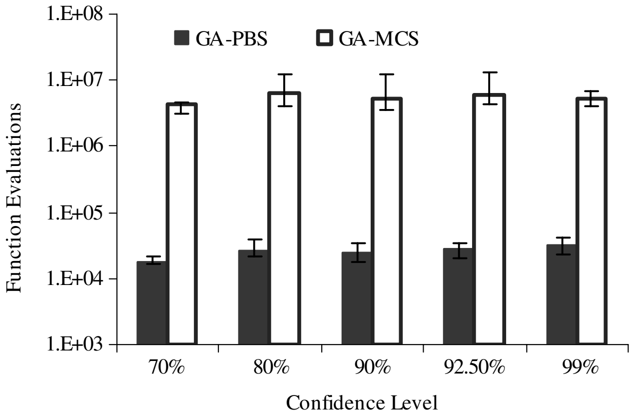

Fig. 4 Comparison of computational cost of MCS and PBS at various confidence levels.

Tables 3 and 4 present additional information about these GA runs. Table 3 shows the mean values of lowest spacecraft mass, computational time, and number of function evaluations computed for 10 runs at each confidence level.

Table 4 shows the expected values of aggregate spacecraft reliability and payload reliability along with the measured probabilities of meeting a specified reliability limit. In the case of the PBS approach, the measured probabilities are computed after the GA run has finished using data from all samples gathered for the spacecraft design over the run of the GA. The values in Table 4 are computed for the lowest mass design from the 10 runs at each confidence level. For the computational times listed for these studies, and in subsequent studies in this paper, a serial genetic algorithm was used, providing a consistent comparison of computational time. Because the GA and MCS are well suited to course-grain parallelization, distributed computational environment could provide significant run time benefits.

1. Comparison of Effectiveness (Solution Quality)

With the PBS approach requiring 2 to 3 orders of magnitude less computational effort than MCS, establishing that the PBS approach finds designs of comparable quality as designs found using MCS is important. The mean spacecraft mass values found from both approaches are very similar, as shown in Fig. 3 and Table 3, indicating that both approaches repeatedly found designs in the same area of the design space at each confidence level. The difference between the maximum and minimum mass of the 10 runs conducted for each approach at each confidence level is small compared to the total spacecraft launch mass (ranging from a low difference of 0 kg to a high difference of 39.3 kg for the MCS approach and a low of 16.7 kg to a high of 43.7 kg for the PBS approach). This small variability in final solutions is common in GA applications and suggests these results are repeatable.

Figure 3 also shows that the minimum spacecraft mass value increases as the confidence level constraint limit increases. The design found using a 70% confidence level constraint by both the MCS and PBS approaches has a launch mass of 3488.17 kg, whereas both approaches converged to a 3602.81 kg design at the 99% confidence level. Table 5 presents expected reliability and probability of success values for these designs. At each of these

Table 3 Mean values of solution quality and computational cost metrics for 10 GA-MCS and 10 GA-PBS runs using several confidence level limits

Confidence level limit	Sampling approach	Mass of best design, kg	No. of generations	Function evaluations	Computational time, h
70%	MCS	3488.17	51	4,264,000	9.79
	PBS	3501.37	82	13,743	1.03
80%	MCS	3551.46	77	6,412,400	15.41
	PBS	3554.56	114	18,926	1.49
90%	MCS	3582.38	63	5,239,800	12.37
	PBS	3592.43	135	22.320	1.76
92.5%	MCS	3602.89	74	6,133,600	13.67
	PBS	3596.23	144	23,714	2.01
99%	MCS	3614.97	61	5,116,800	11.78
	PBS	3609.48	165	27,208	2.57

Table 4 Computed expected values of reliability and probability of success of best design obtained by GA-MCS and GA-PBS at several confidence levels

Confidence level limit	Sampling approach	$E(R_{\text{payload}})$ limit = 99%	$E(R_{\text{spacecraft}})$ limit = 95%	$P(R_{\text{payload}} \geq 99\%)$ measured	$P(R_{\text{spacecraft}} \geq 95\%)$ measured
70%	MCS	99.41%	95.56%	89.06%	71.80%
	PBS	99.53%	95.80%	90.26%	71.76%
80%	MCS	99.60%	95.87%	96.36%	82.58%
	PBS	99.74%	96.02%	96.78%	81.80%
90%	MCS	99.59%	96.51%	97.10%	93.16%
	PBS	99.51%	97.43%	92.97%	98.60%
92.5%	MCS	99.54%	97.09%	95.72%	98.52%
	PBS	99.47%	97.37%	94.10%	98.68%
99%	MCS	99.63%	97.21%	99.46%	99.34%
	PBS	99.64%	96.78%	99.02%	99.07%

Table 5 Computed expected reliabilities and probability of success for minimum mass spacecraft at confidence level constraints of 70 and 99%

Confidence level limit	Spacecraft mass, kg	$E(R_{\text{payload}})$ limit = 99%	$E(R_{\text{spacecraft}})$ limit = 95%	$P(R_{\text{payload}} \geq 99\%)$ measured	$P(R_{\text{spacecraft}} \geq 95\%)$ measured
70%	3488.17 kg	99.42%	95.57%	89.60%	72.80%
99%	3602.81 kg	99.59%	97.22%	99.80%	99.80%

Table 6 Description of design variables of minimum mass solution (3602.81 kg) obtained by MCS and PBS at a confidence level limit of 99%

	Design variable	Discrete values
1	Launch vehicle choice from 8 options	Delta
2–9	HPA type for the 8 C-band Txs	SSPAs for the 1st and 3rd–8th Txs and TWTAs for the 2nd Tx
10	Solar array cell type	Si cells
11	Battery cell type	NiH_2
12	N/S thermal radiator panels coupling	No
13	N/S STK thruster technology	Plasma
14	E/W STK thruster technology	Bipropellant
15–23	Ku and C-band Txs redundancy level	10 total redundant HPAs
24	Propulsion subsystem redundancy level	Duplicate redundancy
25	ADCS redundancy level	Duplicate redundancy
26	TCR subsystem redundancy level	Duplicate redundancy
27	Solar array redundancy level	6% of solar array area

Table 7 Summary of minimum mass designs found by 10 runs of GA-MCS and GA-PBS

	Mass of best design, kg		
	3602.81	3619.48	3624.37
Times found by GA-MCS	3	6	1
Times found by GA-PBS	6	4	0
Expected $R_{payload}$	97.59%	99.64%	99.59%
Expected $R_{spacecraft}$	97.22%	97.29%	97.24%
Measured $R_{payload}$	99.80%	99.40%	99.60%
Measured $R_{spacecraft}$	99.80%	99.40%	99.80%
Differences from Table 6 design	——	Additional redundant 4th C-band Tx HPA	TWTA for 1st C-band Tx, SSPA for 2nd

confidence levels, the expected reliability values exceed the limits and the probability of exceeding the reliability limit exceeds the confidence level.

Table 6 summarizes the technology choices and redundancy levels for the 3602.81 kg design obtained at 99% confidence level. The design obtained at 70% confidence level is similar to the 3602.81 kg design except for the redundancy levels in two subsystems; this accounts for the 115 kg difference in mass. The 3602.81 kg design has a Ku-band repeater with 14 available high-power amplifiers (HPAs), while the 3488.17 kg design has only 13. The 3602.81 kg design has a fully redundant propulsion subsystem with two thruster sets, while the 3488.17 kg design has no redundancy.

Table 7 summarizes the best designs found by the 10 runs of the GA-MCS and GA-PBS approaches using 99% confidence level limits. These three different solutions listed in Table 7 have only minor differences in the discrete design variable choices, and their expected and measured reliabilities are quite comparable. At other confidence level limits, the PBS and MCS approaches exhibit similar behavior. From these comparisons, the PBS approach appears equally effective as the MCS in finding designs under uncertainty that have good objective function values and meet the probability constraints.

2. *Comparison of Efficiency (Computational Cost)*

Figure 4 shows the minimum, mean, and maximum values of the number of function evaluations (as a measure of computational cost) as shown at each confidence level for both the MCS and PBS approaches. Table 3 shows the average number of generations used by the MCS and PBS approaches to reach the stopping criteria and the corresponding computational cost and time. For PBS, the number of function evaluations is the number of generations (plus the initial zeroth generation) multiplied by the population size. For MCS, the number of function evaluations is the number of generations multiplied by the population size multiplied by the number of samples (500). The MCS approach has a large computational cost because each fitness evaluation requires 500 function evaluations for both poor and good designs, while the PBS approach allows for the accumulation of at least 500 samples for the best design from the whole run of the GA. The ratio of the average computational cost of MCS to that of PBS is 310, 339, 235, 259, and 188 at the confidence level constraints of 70, 80, 90, 92.5, and 99%, respectively. As the confidence level limit increases, corresponding to more stringent probability constraints, the cost ratio decreases from about 300 to just below 200. Although this indicates 2 orders of magnitude faster performance for PBS over MCS at the largest confidence level limit of 99%, the performance advantage of PBS decreases slightly as the confidence level limit increases.

Table 3 also includes the computational time required by both approaches. At the 99% confidence level, the MCS runs lasted 11.8 h and used 5,116,800 function evaluations on average, while the PBS runs lasted 2.57 h and used 27,207 function evaluations on average until convergence. The PBS approach incurred only 0.53% of the computational cost and 22% of the computational time of the equivalent MCS runs.

Increasing the number of samples required for the PBS stopping criteria from 500 to 5000 at a confidence level of 99% resulted in average run times of 3.62 h and average cost of 31,422 function evaluations. This is only 0.61% of the computational cost and 31% of the time needed for the MCS runs with only 500 samples.

Sorting the GA population in equivalence classes, rather than fitness evaluation, consumes most of the computational time associated with PBS. The PBS runs here used basic sorting approaches. Each new design is compared to all the designs in its population and also to all the populations from previous generations. This basic sorting technique is performed to collect samples from all designs sharing the same deterministic design variables. Advanced sorting using dynamic memory approaches may significantly reduce the PBS computational time to make it directly proportional to the number of fitness evaluations.

B. Analysis of the PBS Approach

Two issues related to the PBS approach need further exploration. The first is the rate at which individual designs accumulate samples during the GA search. Initially, the GA population is randomly distributed throughout the deterministic design space and, therefore, provides very few samples, if any, for each design. It may require many generations before the population starts congregating in promising regions of the design space. Toward the end of the run, the GA population may have numerous copies of the same deterministic design. To enact the stopping criterion for the PBS approach, the best solution must have accumulated at least the required number of samples throughout the GA run.

Figure 5 was generated by considering the best design obtained from one run of the GA-PBS approach using 5000 samples as the minimum number required by the stopping criterion and using 99% confidence level limits. Ten runs were conducted at this confidence level with the 5000 sample stopping criterion, and all resulted in the same spacecraft design. The plot in Fig. 5 shows the number of samples obtained during each generation for this best design from one run. This design first appeared in the population as one out of 164 individuals in the 67th generation. At this point, the design had one sample for its uncertain parameters. Figure 5 shows that between one and four samples were added to each generation from the 67th through the 115th generation, because between one and four copies of this chromosome are in the population during each of these generations. As this best design begins to propagate through the population, the number of samples provided per generation increases from nine samples in the 116th generation to 133 samples in the

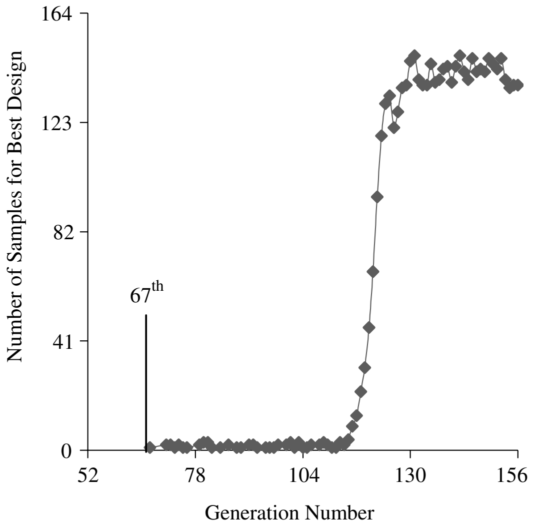

Fig. 5 Number of samples contributed from each generation to the best design chosen by the GA using the PBS approach for a 5000 sample convergence criteria.

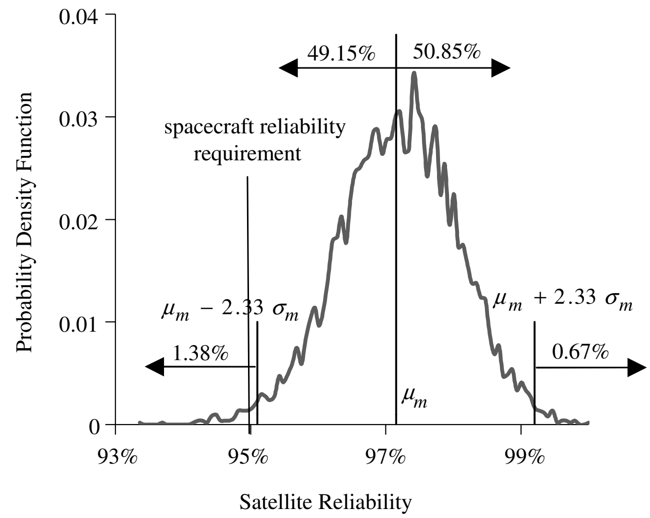

Fig. 6 Measured probability density function for a spacecraft reliability limit of 95% and a confidence level of 99% using PBS.

125th generation. The GA then stabilizes its sample addition rate at a plateau adding about 140 samples per generation, until the 5000 sample criterion is met. During these last generations, nearly 85% of the 164 individual designs have the same chromosome. This design could obtain more samples, which would improve accuracy of predicted reliabilities, if the GA ran for more generations. The last generation also maintains some population diversity; 27 individuals differed from the best design. This indicates that if a better design were available, the GA still had the possibility to find it.

The second PBS approach issue is a comparison of the postprocessed reliability probability distribution to the assumed Gaussian distribution used to construct surrogate confidence level constraints. The samples accumulated in the PBS run provide a measured mean μ_m and a measured standard deviation σ_m for use as the mean and standard deviation of an assumed Gaussian distribution. This assumption of a Gaussian distribution and the corresponding surrogate constraint formulation allows the GA to proceed when the number of samples is very small. After the run is complete, probabilities of meeting or exceeding the reliability value limits can be computed because the number of samples is high for good designs. Table 8 shows the mean values of the actual calculated probabilities of success for the five confidence levels investigated in this effort using 5000 samples for the PBS runs. The postprocessed probabilities exceed the confidence level constraint limits in all cases.

The postprocessed probability distribution of spacecraft reliability for the best design obtained from one of the 10 PBS runs using 5000 samples in the stopping criterion and with 99% confidence level limits appears in Fig. 6; this is the same design whose sample addition rate appears in Fig. 5. This best design has a launch mass of 3602.81 kg with an expected spacecraft reliability value of 97.15% and an estimated standard deviation of 0.88%. Here, the confidence level surrogate constraint was intended to enforce the probability that the aggregate spacecraft reliability value exceeds 95%, that is, $P(R_{\text{spacecraft}} \geq 95\%) \geq 99\%$. This could also be interpreted as enforcing that 99% (4950) of the 5000 samples have spacecraft reliability values greater than 95%.

Table 8 Mean computed probability of success of meeting spacecraft reliability limits for best design obtained by the GA-PBS using 5000 sample runs

Confidence level limit	$P(R_{\text{payload}} \geq 99\%)$ measured	$P(R_{\text{spacecraft}} \geq 95\%)$ measured
70%	90.11%	74.14%
80%	98.96%	81.16%
90%	92.98%	97.61%
92.5%	94.23%	98.74%
99%	99.02%	99.07%

The postprocessed probability density function of spacecraft reliability in Fig. 6 has a computed 98.98% probability that the spacecraft reliability exceeds 95%. This is less than the imposed confidence level surrogate constraint of 99%, so the surrogate constraint was satisfied, but the desired constraint is slightly violated. This is the result of only one run; the mean of 10 runs with the same confidence level and sample number is 99.07% (see Table 8). The distribution in Fig. 6 generally follows a Gaussian distribution, but the center of the computed distribution is not exactly at its mean. This offset in the aggregate spacecraft reliability distribution can be attributed to the pseudo-Gaussian distributions for component and subsystem reliabilities, in which the maximum possible reliability value is one. The offset could be the reason why the computed probability of 98.98% is less than the surrogate constraint confidence level limit of 99%. Additionally, when reevaluating this design in a postprocessing run with 50,000 samples to provide more accurate estimates, the estimated spacecraft reliability was 97.2% with 49,531 out of the 50,000 samples producing feasible designs. Therefore, a more accurate measured probability of success, or probability of meeting the reliability requirements, for this design is 99.1% $(49,531/50,000)$.

Based on the above results, it appears that the Gaussian distribution assumption used in the surrogate constraint formulation is reasonable, but perhaps it is less accurate than desired. Section VII of this paper extends PBS to cases where the uncertain parameters belong to non-Gaussian or a mix of Gaussian and non-Gaussian distributions.

V. Testing for Scalability

Four versions of the spacecraft optimization problem are considered to investigate how the PBS approach scales with increasing problem size. The problem presented previously in this paper, and in [1], is the largest version, with 27 design variables, and six design constraints (three deterministic and three uncertain). Three smaller problems are variations on the spacecraft optimization problem, with fewer design variables and constraints. Table 9 compares the sizes of the four problems, where the largest problem is designated as the "fourth" problem.

The third problem eliminates the choice of the launch vehicle as a design variable and the constraint on the launch vehicle reliability, G_4. Delta was the launch vehicle modeled in this formulation.

The second problem includes the launch vehicle selection as a design variable but fixes the payload design, eliminating design variables 2–9 [1] and 15–23 [1], which are listed in Table 1, and

Table 9 Description of the spacecraft design problems investigated to test the effect of problem scalability on the performance of the optimization algorithms

Problem designation	Design variables	Encoding bits	Deterministic constraints	Uncertain constraints
First	9	13	3	1
Second	10	16	3	2
Third	26	38	3	2
Fourth	27	41	3	3

Table 10 Payload description for first and second problem formulations

Payload parameter	Description
C-band repeaters HPA type	SSPAs for repeaters 1 and 3–8, TWTAs for repeater 2
Ku band and C band repeaters redundancy	A total of 10 redundant HPAs: 2 for the Ku band repeater and 1 for each of the 8 C-band repeaters

decreasing the number of design variables to 10. The uncertain constraint on payload reliability G_5 does not appear in this formulation. Table 10 summarizes the repeater design used for this problem. This payload repeater design was part of the optimal spacecraft design found in the previous studies of [1] and described in Table 6.

The first problem formulation uses a fixed launch vehicle and payload repeater design, eliminating design variables 1–9 [1] and 15–23 [1] and reducing the number of design variables to nine. The uncertain constraints on launch vehicle reliability and payload reliability do not appear. Delta served as the launch vehicle and the repeater choices described in Table 10 served as the payload.

Each of the four problems were run 10 times (to assess repeatability) using the DMB approach [1,2], the MCS approach, and the PBS approach to address uncertain constraints. The reliability constraints used a confidence level of 92.5%, because this is the highest confidence level at which the DMB approach generated feasible designs. MCS used 500 samples for each fitness evaluation and PBS used 500 samples in the stopping criterion. An equal sample size for these methods allows for a comparison of computational cost of the two approaches with the same level of accuracy in the predicted reliability values. A larger sample size for the MCS approach would be computationally prohibitive. As a computational cost comparison, the number of function evaluations required until convergence for all three approaches appears in Fig. 7 using a logarithmic scale. The plot includes the minimum and maximum number of function evaluations as error bars above and below the mean number of evaluations for all four problems.

For all four problem sizes, the computational cost of PBS is only slightly higher than that of the DMB approach (which does not use sampling, but instead uses a safety-factor approach for uncertainty) using the GA, and the computational cost of PBS is significantly lower than that of MCS. A summary of results for all the first, second, and third problem sizes appears in Table 11. Results of the fourth problem previously appeared in Table 3. The safety-factor-like approach of DMB appears to overdesign the spacecraft. Both PBS and MCS repeatedly found similar quality, feasible design solutions that satisfy the constraints with much lower launch mass than the

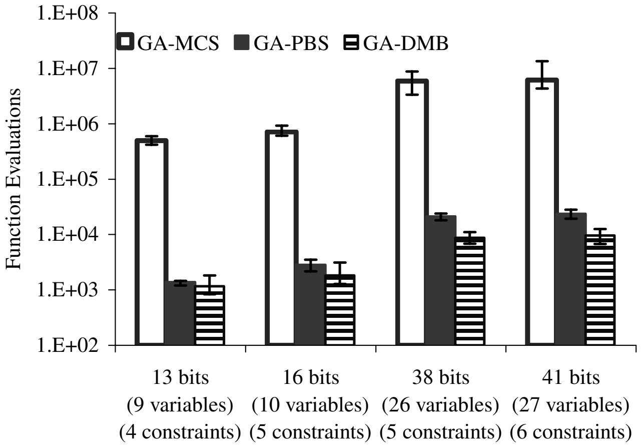

Fig. 7 Comparison of the computational cost of DMB, PBS, and MCS approaches for multiple problem sizes.

DMB approach designs, indicating that these sampling approaches more appropriately handle uncertainty.

For the four problem sizes, the PBS approach's computational cost is 2 orders of magnitude less than that of the MCS approach, with the same number of samples used for final predictions of uncertain parameters. The computational cost of both PBS and MCS grows exponentially with the increase in problem size, but the cost associated with PBS remains a fraction ($\approx 1/300$) of the MCS cost. This computational cost reduction appears to be an important contribution of the PBS approach.

The significantly lower computational cost is due to the fact that the PBS approach accumulates the required 500 samples for the best design solution during the optimization run, whereas MCS uses 500 samples for each fitness evaluation—for poor as well as good designs. For the largest spacecraft design problem, the PBS approach repeatedly found an optimal design (the same design found via MCS) that satisfies the reliability requirements at high confidence levels in about 2 h (compared to 13.7 h for MCS). To the authors' best knowledge, the largest (fourth) problem investigated here is the largest spacecraft system design problem under deterministic or uncertain conditions (based upon number of design variables and number of constraints) investigated in the literature [8,16,17]. Larger problem sizes for other complex systems could benefit from the PBS approach.

VI. Variable Population-Based Sampling (VPBS)

The VPBS approach presents a compromise between MCS and PBS with the objective of further reducing the computational cost of discrete optimization under uncertainty. The results in Tables 3 and 11 show that the PBS approach requires 2 orders of magnitude fewer function evaluations than the MCS approach, but these results also show that MCS reaches its stopping criterion in fewer generations (roughly 1/3 of PBS). PBS provides many samples for designs with good fitness values and a few samples for poor-performance designs. However, the GA-PBS evolves through a large number of generations until the best design acquires the specified minimum number of samples. The example in Fig. 5 shows that the best design first appears in the 67th generation, but does not accumulate a significant number of samples until the 116th generation. Also in this example, the GA-PBS continues to evolve from generation 125 to

Table 11 Comparison between DMB, PBS, and MCS for the first, second, and third problem sizes

	First problem			Second problem			Third problem		
Evaluation criterion	DMB	PBS	MCS	DMB	PBS	MCS	DMB	PBS	MCS
Mass, kg	4623	3602	3,602	4719	3602	3,602	4743	3,604	3,602
Generations	21	25	18	27	43	21	56	138	77
Function evaluations	1159	1383	496,600	1830	2848	713,600	8770	21,158	5,905,200
Time, h	0.005	0.009	1.069	0.003	0.033	1.395	0.035	1.495	13.510

generation 156 as the best design is accumulating samples to reach the stopping criterion, but there is little design space search occurring during these generations.

The concept of the VPBS approach maintains the idea of accumulating samples of uncertain properties associated with multiple occurrences of the same design **x**. However, in VPBS, rather than adding only one sample for each occurrence of **x** in the GA's population, the number of samples associated with each design starts at one when the quality of most designs is poor early in the GA, and the number of samples associated with each occurrence of a given design increases over successive generations. If this number of samples for each occurrence increases as the population starts to congregate at and near the eventual best design, then this best design would accumulate the required minimum number of samples in fewer generations than in the basic PBS approach and perhaps in fewer function evaluations.

Three cases investigate the VPBS approach. The objective is to identify the variable sampling rate scheme that yields the lowest number of function evaluations, which is intended to be lower than that of the PBS approach. In the first case, VPBS-I, each individual in the population during the first 10 generations receives only one sample, while each individual in the population during the second set of 10 generations receives two samples, and so on with one additional sample added every 10 generations. In the second case, VPBS-II, each individual in the population receives only one sample during the first 20 generations, and one additional sample per individual is added every 20 generations. Finally, the third case, VPBS-III, starts with one sample per individual and adds one additional sample per individual every 30 generations.

Each case used confidence level limits of 99% in the surrogate constraints, and the code stopped when the minimum fitness value did not change for more than ± 0.01 kg for 10 consecutive generations and when the minimum fitness design had accumulated 5000 samples. To assess repeatability of the results, each method was run 10 times. Table 12 summarizes the number of generations and function evaluations associated with the VPBS runs and the previously presented 5000-sample PBS runs.

All four approaches found the same spacecraft design for all 10 runs performed for each approach. This design is the same 3602.81 kg spacecraft described in the previous section, so using the VPBS approach has not changed the quality of the solution. All three VPBS cases reached their stopping criterion in fewer generations than the PBS runs. The number of generations evaluated in the VPBS cases increased as the number of generations between the increments in samples per individual increased. The VPBS-I approach added an additional sample per occurrence every 10 generations and these runs required a mean of 68 generations, while the VPBS-III approach added an additional sample per occurrence every 30 generations and required a mean of 78 generations.

In terms of total computational cost, the VPBS-I case required more function evaluations than the basic PBS approach, the VPBS-II case matched the number of function evaluations of the basic PBS approach, and finally, the VPBS-III case required fewer function evaluations than the basic PBS approach. This simple comparison suggests that perhaps the number of samples given to each occurrence of a given design in each generation could be a function of the homogeneity of the GA population, that is, the fewer different individual designs appear in the population, the larger the number of samples should be given to each design to accumulate the required minimum sample size faster.

Table 12 Comparison between PBS, VPBS-I, VPBS-II, and VPBS-III using 5000 sample runs at 99% confidence level

	No. of generations			No. of function evaluations		
Sampling method	min	mean	max	min	mean	max
PBS	138	191	245	22,796	31,422	40,344
VPBS-I	65	68	70	41,328	44,887	47,068
VPBS-II	64	74	84	22,960	29,799	36,900
VPBS-III	72	78	87	21,156	23,862	28,536

VII. Generalized Population-Based Sampling (GPBS)

In the above PBS investigations, the pseudo-Gaussian distributions described the uncertain parameters (component and subsystem reliability). Therefore, the assumption that the aggregate system reliability function followed a normal distribution seemed appropriate. However, uncertain properties in general may follow non-Gaussian distributions or may be a mix of Gaussian and non-Gaussian distributions. The surrogate constraint formulation developed for the PBS approach relied upon the assumption of aggregate Gaussian distributions, because calculating, rather than estimating, actual probabilities of success is problematic at the beginning of the search process when only one sample is available.

The GPBS approach is another variant of PBS that uses a small, fixed number of samples for fitness evaluation of every individual design to allow for a crude estimation of probability, even in early generations of the GA. Accumulating samples over the run of the GA, as in the basic PBS approach, allows for accurate calculation of probabilities as the GA reaches its stopping criterion. With a small number of samples for fitness evaluation in early generations, probabilities of success can be computed from uncertain properties with any type of distribution, albeit with limited accuracy. Using more than one sample for each occurrence of an individual every generation increases the computational cost beyond that of the basic PBS approach, so this number of samples should remain low. In the GPBS approach, the confidence level constraints use a counting approach to calculate probabilities of success.

In this investigation, each occurrence of a design in the GA receives three samples of $\boldsymbol{\xi}$. Samples are also accumulated from the population throughout the run of the GA as in the basic PBS approach. Because each individual has N_{samples} of at least three, designs in the initial population (where it is most likely that each individual is a different design) can have computed probabilities of 0.00, 0.33, 0.67, or 1.00. The accuracy of this prediction is low, but these four possible values of probability provide better discrimination between nearly feasible and definitely infeasible designs than in the basic PBS approach. Also in this investigation, a mix of gamma, lognormal, and Gaussian distributions represented the uncertain reliability of components and subsystems with published failure rates used as the means of these distributions. Details on the values used for component and subsystem reliability distributions are available in [2].

The GPBS approach was compared to the MCS approach for solving the uncertain spacecraft design problem at confidence level constraint limits of 99%. The MCS used the same mix of gamma, lognormal, and Gaussian distributions for the satellite component and subsystem reliability distributions. The stopping criterion for the GPBS approach halts the GA if the minimum fitness value does not change more than ± 0.01 kg for 10 consecutive generations and if the best design has accumulated 500 samples. The MCS approach used 500 samples for each function evaluation and stopped when the fitness function did not change for more than ± 0.01 kg for 10 consecutive generations. Table 13 summarizes the GA runs with the GPBS and the MCS approaches.

Both MCS and GPBS generate solutions in the same area of the design space that satisfy reliability limits with the required probabilities of success. The computational cost of the GPBS

Table 13 Comparison of GPBS and MCS using a mix of non-Gaussian distributions for components and subsystems

	Sampling method	
Evaluation criteria	GPBS	MCS
Launch mass, kg	3612.79	3602.81
No. of generations	82	49
Function evaluations	40,834	4,100,000
R_{payload}	99.60%	99.59%
$R_{\text{spacecraft}}$	97.14%	97.15%
$P(R_{\text{payload}} \geq 99\%)$	99.29%	99.40%
$P(R_{\text{spacecraft}} \geq 95\%)$	99.62%	99.40%

approach is 2 orders of magnitude less than that of the MCS approach, based on the number of function evaluations. When the GPBS approach run continued its search beyond the initial stopping criterion, it could accumulate 5000 samples for the best design in only five more generations (i.e., a total of 87 generations), because the best design dominates the population and aggressively accumulates samples near the end of the run. In this example, the rate of sample addition to the best design is 3 times as aggressive as that of the PBS approach, because each occurrence of a given design receives three samples every generation. This allows the GPBS approach to converge in fewer generations than PBS, but with a comparable number of function evaluations without the need for the normality assumption for the aggregate uncertain metric.

VIII. Conclusions

This paper describes versions of a genetic algorithm (GA) developed to solve discrete optimization problems in the presence of uncertainty. This paper presented studies using the example application of a spacecraft conceptual design problem that seeks to minimize the launch mass of the spacecraft using 27 discrete design variables describing component and subsystem technology choices, redundancy levels, and the choice of the launch vehicle. This problem involves uncertain component failure rates and constraints on the probability of successfully achieving prescribed system reliability values.

Population-based sampling (PBS), a GA-based approach presented here, allows for probabilistic evaluation of performance constraints without a considerable increase in the computational cost beyond what is required for a deterministic GA. A comparison of the GA-PBS approach with the state-of-practice approach of coupling Monte Carlo sampling (MCS) with a GA for design under uncertainty provided an assessment of the efficiency (computational cost) and effectiveness (solution quality) of GA-PBS. Both approaches provide similar quality solutions at all imposed confidence levels, and the computational cost associated with PBS was 2 orders of magnitude less than that of the MCS approach. The computational cost of MCS seems prohibitive for this application, which includes a large number of design variables. PBS provided a comparatively inexpensive approach for incorporating uncertainties in the spacecraft conceptual design problem. Alternatively, it can be concluded that the PBS approach allows for far greater accuracy, which leads to higher confidence levels, and still uses many fewer function evaluations than the MCS approach.

The performance of the PBS approach was compared to both the MCS approach and a safety-factor-like deterministic margin-based (DMB) approach using three additional versions of the spacecraft design problem, each of which has a different number of design variables and constraints. This comparison demonstrated that for varying problem sizes, the probabilistic approaches, MCS and PBS provide spacecraft design solutions that satisfy the required reliability constraints at high confidence levels but have launch mass values that are 1000 kg less than the solutions provided by the DMB approach. At all problem sizes and all confidence levels, the PBS approach requires about 1/300th of the function evaluations required by the MCS approach.

Variable population-based sampling (VPBS), a variant of the basic PBS approach, reduces the computational cost for discrete optimization under uncertainty below that of PBS. The modification of the VPBS approach from the basic PBS is the gradual increase of the number of samples for each occurrence of a given design in the GA run. The VPBS approach results indicate that it requires fewer generations and function evaluations than that of the basic PBS approach to obtain the same solution.

Finally, the generalized population-based sampling (GPBS) approach allowed for uncertain parameters with non-Gaussian distributions. The GPBS approach directly calculates probabilities of success to enforce confidence level constraints using a small number of samples for poor fitness designs and a large number of accumulated samples for good-fitness designs. The investigation of GPBS using a mix of gamma, lognormal, and Gaussian distributions for uncertain parameters indicates that the computational cost is on the same order of magnitude as that of the basic PBS to obtain solutions of similar quality as the more rigorous MCS approach.

In conclusion, the computational times associated with PBS and its variant approaches, which are on the order of few hours of turnaround time using a serial GA, would enable discrete optimization under uncertainty for conceptual design optimization efforts.

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